Anglo-Chinese School
(Barker Road)

MID-YEAR EXAMINATION 2018

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC

ADDITIONAL MATHEMATICS 4047
PAPER 1

2 HOURS

Additional Materials: 8 Answer Papers

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examinations, fasten your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Answer all the questions

1 It is given that $\int f(x)dx = \tan x + k\cos x + c$, where c is an arbitrary constant of integration, and that $\int_{\pi}^{2\pi} f(x)dx = 1$.

(i) Find k. [2]

(ii) Find the expression for $f(x)$. [3]

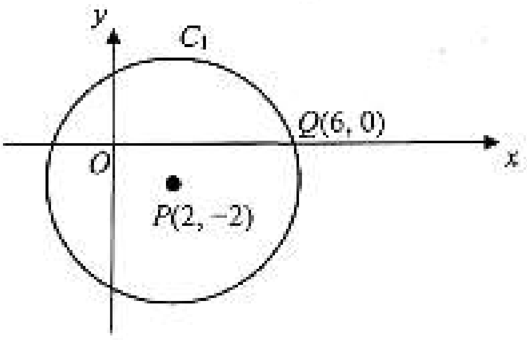
2 Given that $\tan A = -\frac{24}{7}$, $\cos B = \frac{12}{13}$ and the angles A and B are in the same quadrant, find without using a calculator, the value of

(i) $\operatorname{cosec} A$, [2]

(ii) $\tan 2B$, [2]

(iii) $\sin\left(\frac{A}{2}\right)$. [2]

3 The diagram, which is not drawn to scale, shows circle C_1.



Circle C_1 has its centre at centre at $P(2, -2)$ and passes through $Q(6, 0)$.

(i) Find the equation of circle C_1. Express your answer in the form of $x^2 + y^2 + 2gx + 2fy + c = 0$. [3]

(ii) Another circle, C_2, has equation $x^2 + y^2 - 12x - 10y + 36 = 0$. By considering the centre of C_2, explain if C_2 intersects the y-axis. [3]

4 Given $\sin\left(2y + \frac{\pi}{3}\right) = 2\cos 2y$,

(i) show that $\tan 2y = 4 - \sqrt{3}$, [3]

(ii) solve $\sin\left(2y + \frac{\pi}{3}\right) = 2\cos 2y$ for $0 \le y \le \pi$. [3]

5 Water is poured into a bucket at a rate of 24π cm^3/s. The volume, V cm^3, of water in the bucket is given by $V = \dfrac{\pi x^2(15-x)}{3}$, where $x > 4$. The depth of the water in the bucket is x cm. Find

(i) the rate of increase in the depth when $x = 8$, [3]

(ii) the volume of water at the instant when the rate of increase in the depth is 1 cm/s, leaving your answer in terms of π. [4]

6 The number of insects, N, in a colony after t days can be modelled by $N = 1000e^{at}$, where a is a constant. There are 8000 insects after 5 days.

(i) Find the initial number of insects in the colony. [2]

(ii) How many insects are there after 20 days? Correct your answer to 2 significant figures. [3]

(iii) Hence, sketch the graph of $N = 1000e^{at}$ for the first 5 days [2]

7 It is given that $f(x) = |x^2 - 6x + 5|$ for $-1 \le x \le 7$.

(i) Sketch the graph of $y = f(x)$, indicating the intercepts of both axes and the coordinates of the turning point. [3]

(ii) State a possible integer value of k for which the equation $|x^2 - 6x + 5| = k$ has 4 solutions. [1]

(iii) Solve $|x^2 - 6x + 5| = 5$. [2]

(iv) Hence, or otherwise, state the set of values of x for which $|x^2 - 6x + 5| > 5$ in the given range. [2]

8 (a) The quadratic equation $2x^2 - 9x + 4 = 0$ has roots α and β. Find the quadratic equation whose roots are $\alpha\beta^2$ and $\alpha^2\beta$. [5]

(b) Given that α is a root of the equation $2x^2 = 3x - 4$, show that $4\alpha^3 = \alpha - 12$. [3]

9 **(a)** The equation of a curve is $y = \frac{3x+4}{x-2}$, where $x \neq 2$.

Determine whether y is an increasing or decreasing function. [3]

(b) The curve $y = (2x-1)^4$ has a stationary point M. Determine the nature of point M. [5]

10 A piece of wire, 160 cm long, is divided into two parts to form two disjoint shapes. One part is bent to form an equilateral triangle of sides x cm. The other part is bent to form a sector of a circle of radius x cm .The angle subtended by the radii of the sector is θ radian.

[For a sector of radius r, the arc length is $r\theta$ and the area is $\frac{1}{2}r^2\theta$.]

(i) Express θ in terms of x. [2]

(ii) Show that the total area, A cm^2, enclosed by the two shapes is given by

$A = 80x + \frac{x^2}{4}(\sqrt{3} - 10)$. [2]

(iii) Given that x and A can vary, find the value of x for which A has a stationary value. Determine whether it is a maximum or a minimum area. [5]

11 The equation of a curve is $y = (x-3)\sqrt{2x+2}$. The point P lies on the curve and has an x-coordinate of 1. The tangent to the curve at P meets the x-axis at A and the y-axis at B.

(i) Find the coordinates of A and of B. [6]

(ii) The point Q also lies on the curve. The normal to the curve at Q is parallel to the tangent of the curve at P. Find the x-coordinate of Q. [4]

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MID-YEAR EXAMINATION 2018

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC

ADDITIONAL MATHEMATICS 4047
PAPER 2

2 HOURS 30 MINUTES

Additional Materials: 10 Answer Papers
1 Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

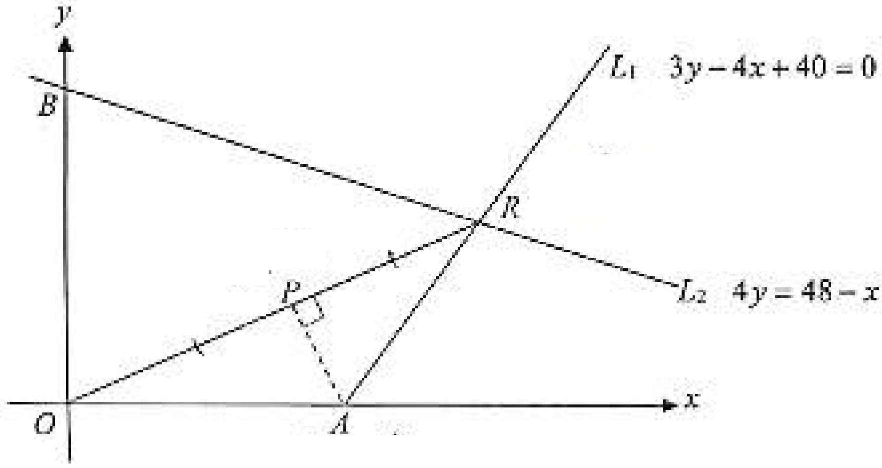
At the end of the examinations, fasten your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Answer **all** the questions

1 **(i)** Prove that $\sin 2\theta - \tan\theta\cos 2\theta = \tan\theta$. [3]

(ii) Hence, without the use of calculator, show that $\tan 67.5° = \frac{\sqrt{2}}{2-\sqrt{2}}$. [4]

2 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows the lines L_1 and L_2 intersecting at R. Line L_1 has equation $3y-4x+40=0$ and L_2 has equation $4y=48-x$. The point P is the midpoint of OR. The line L_1 intersects the x-axis at A and the line L_2 intersects the y-axis at B.

(i) Show that angle $APR = 90°$. [5]

(ii) Find the area of the quadrilateral $OARB$. [2]

3 The table shows experimental values of two variables x and y, which are connected by an equation of the form $y = x\ln(q+px)$ where p and q are constants.

x	0.5	1	1.5	2	2.5
y	0.896	1.61	2.08	2.20	1.73

(i) Using a scale of 4 cm to 1 unit for the horizontal axis and 2 cm to 1 unit for the vertical axis, plot $e^{\frac{y}{x}}$ against x to obtain a straight line graph. [2]

(ii) Hence, use your graph to

(a) estimate the value of p and of q, [3]

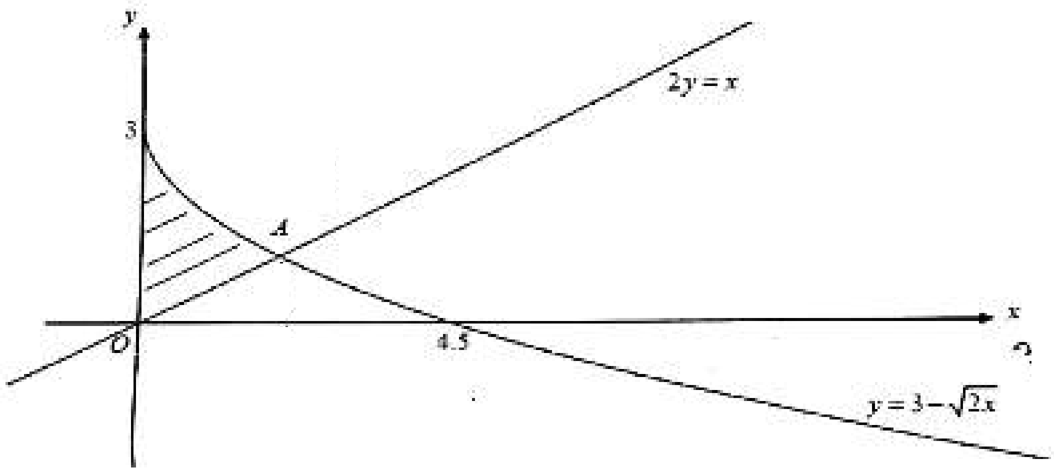
(b) obtain the value of y when $x = 1.2$, correct to 3 significant figures. [1]

(iii) What is the gradient of the straight line when x is plotted against $e^{\frac{y}{x}}$? [1]

4 **(a)** Given that the term independent of x in the binomial expansion of $\left(x^2+\frac{p}{x}\right)^6$ is 1215, find the possible values of p. [4]

(b) The expansion of $(2+3x)\left(1-\frac{x}{2}\right)^n$ in the ascending powers of x as far as the term in x^2 is $2-5x+qx^2$. Find the value of n and of q. [4]

5



The diagram shows parts of the curve $y=3-\sqrt{2x}$ and the line $2y=x$. The curve and the line intersect at the point A. Determine the area of the shaded region. [8]

6 **(a)** Without using a calculator, find the integer values of a and of b for which the solution of the equation $x\sqrt{7}=x\sqrt{2}+\sqrt{32}$ is $\frac{a+b\sqrt{14}}{5}$. [5]

(b) Given that $\frac{(\log_x y)^2}{\log_y x}+64=0$, express y as a power of x. [4]

7 **(a)** Given that $\int_2^7 f(x)\,dx = 8$, evaluate

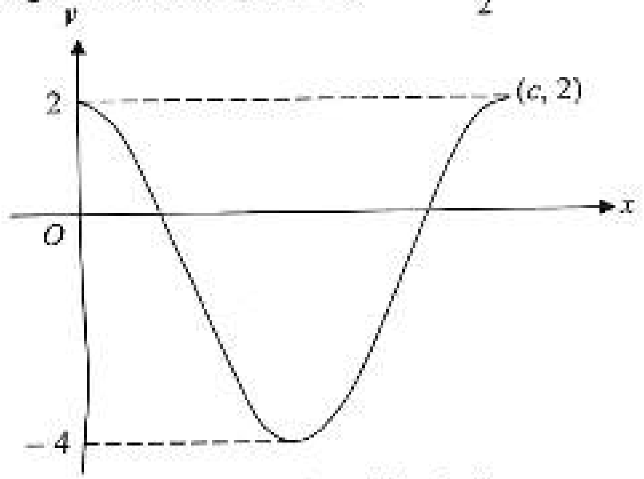
(i) $\int_7^2 f(x)\,dx$ [1]

(ii) $\int_2^7 [2f(x) - 3]\,dx$. [2]

(b) **(i)** $\frac{d}{dx}(x^4 \ln x)$. [2]

(ii) Hence, evaluate $\int_1^3 2x^3 \ln x \; dx$. [4]

8 **(a)** The diagram shows the graph of $y = a\cos\frac{1}{2}x + b$ where x is in radians.



(i) Write down the value of a, of b and of c. [3]

(ii) Find the coordinates of the points where the curve meets the x-axis. [4]

(b) Sketch $y = 3\tan 2x$ for $0° \le \theta \le 180°$. [3]

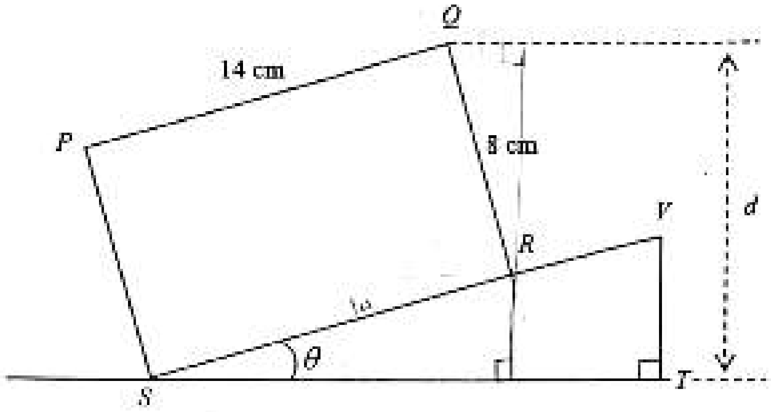
9 **(a)** Determine the set of values of constant k for which the equation $k(x - 2) = x^2 - 3$ has 2 distinct real roots. [3]

(b) The cubic polynomial $f(x)$ is such that the coefficient of x^3 is -2 and the roots of $f(x) = 0$ are 1, p and p^2. It is given that $f(x)$ has a remainder of -14 when divided by $x - 2$.

(i) Show that $p^3 - 2p^2 - 2p - 3 = 0$. [3]

(ii) Hence, find the value of p and show that there are no other real values of p that satisfy this equation. [5]

10



The diagram shows the side view of a rectangular block *PQRS*, with dimensions 14 cm by 8 cm, placed on a ramp, *VS*, tilted at an acute angle of $\theta°$, where $0° < \theta < 90°$. The ramp is placed on a horizontal surface *ST* and *d* is the perpendicular distance from *Q* to *ST*. $\angle VTS = 90°$.

(i) Explain clearly why $d = 8\cos\theta + 14\sin\theta$. [2]

(ii) Express *d* in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) State the maximum value of *d* and find the corresponding value of θ. [3]

(iv) Find the smallest value θ such that $d = 10\sqrt{2}$. [4]

11 A particle moves in a straight line, such that its distance, *s* metres, from a fixed point *O* is given by $s = e^{2t} - 3e^{t} + 7$, where *t* is the time in seconds after passing *O*.

(i) Find the initial position of the particle. [2]

(ii) Show that the particle takes ln(1.5) seconds to first come to instantaneous rest. [3]

(iii) Find the total distance travelled by the particle in 1 second. [4]

(iv) Will the particle ever achieve maximum velocity? Explain clearly with working. [3]

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