# Bukit Batok Secondary School
# First Semestral Examination 2018
Secondary 3 Express

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## ADDITIONAL MATHEMATICS

**4047**
09 May 2018
1215 – 1415
2 hours

Additional Materials: 8 sheets of writing paper
1 sheet of graph paper

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**READ THESE INSTRUCTIONS FIRST**

Write your name, class and register number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Write your answers on the separate answer paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

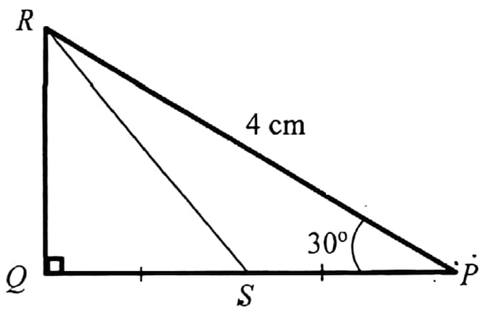
1 A curve has an equation $y = 2x^2 - 3x + c$, where $c$ is an integer.

**(i)** Find the smallest value of $c$ for which the curve is entirely above the $x$-axis. [3]

**(ii)** For $c = -10$, find the range of values of $x$ for which $y \leq 5x$. [3]

2 **(a)** Given that $\sin\theta = -q$ where $90° \leq \theta \leq 270°$ and $q$ is positive, find $\tan\theta$ in terms of $q$. [2]

**(b)**




The diagram shows a triangle $PQR$ where $PR = 4$ cm, angle $RPQ = 30°$ and angle $PQR = 90°$. $PQ = 2PS$ and $PSQ$ is a straight line.

**(i)** Show that $PQ = 2\sqrt{3}$ cm. [1]

**(ii)** Hence, find the exact length of $RS$. [3]

3 The function $f(x) = 2x^3 + ax^2 + 4x + b$, where $a$ and $b$ are constants, is exactly divisible by $x - 3$. Given that it leaves a remainder of $-55$ when divided by $x + 2$,

**(i)** find the value of $a$ and of $b$, [4]

**(ii)** determine, showing all necessary working, the number of real roots to the equation $f(x) = 0$. [4]

4 **(a)** Find the values of $k$ for which line $y = 2x - k$ is a tangent to the circle $x^2 + y^2 = 20$. [4]

**(b)** **(i)** Find the values of $p$ for which the quadratic equation $2px^2 + (8 - 4p)x + p + 1 = 0$ has real roots. [3]

5 The equation $x^2 - 8x + m - 3 = 0$, where $m$ is a constant, has roots $\alpha$ and $\beta$.

**(i)** Find the value of $m$ if $\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{1}{2}$. [4]

**(ii)** Given that $m = 1$,

**(a)** show that $\alpha^3 + \beta^3 = 560$, [2]

**(b)** hence, find a quadratic equation whose roots are $\alpha^3$ and $\beta^3$. [2]

6 **(a)** Solve the equation $5 - |2x - 1| = x$. [3]

**(b)** The graph of $y = \log_a x$ passes through the points with coordinates $(4, -1)$, $(b, 3)$ and $(2, c)$.

**(i)** Determine the value of each of the constants $a$, $b$ and $c$. [3]

**(ii)** Sketch the graph of $y = \log_a x$, indicating clearly the $x$-intercept. [2]

7 **(a)** By using the substitution $u = 3^x$, solve $3(9^x - 12) = 3^{x+1}$. [4]

**(b)** A herd of 20 deer is introduced to an island where there had been no deer before. After $t$ years, the population of deer, $D$, is given by $D = \frac{n}{1 + 4e^{-0.14t}}$, where $n$ is a constant.

**(i)** Show that $n = 100$. [1]

**(ii)** Find the value of $t$ when the population of deer reaches 40. Give your answer correct to the nearest integer. [2]

**(iii)** Explain why the population of deer can never exceed 100. [1]

8 **(i)** Show that $x + 2$ is a factor of $3x^3 + 11x^2 + 8x - 4$. [1]

**(ii)** Factorise completely the cubic polynomial $3x^3 + 11x^2 + 8x - 4$. [3]

**(ii)** Hence, express $\frac{9x^2+2x-18}{3x^3 + 11x^2 + 8x - 4}$ as the sum of 3 partial fractions. [4]

9 (a) Solve $4\log_5 y - \log_{25} y = 7$. [4]

(b) Triangle $ABC$ is right angled at $B$ and its area is $(3\sqrt{5} - 1)$ cm$^2$. Given that $AB = (2\sqrt{5} + 2)$ cm, find, leaving your answers in simplest surds,

(i) the length of $BC$, [3]

(ii) the perimeter of the triangle $ABC$. [3]

10 A curve has the equation $y = 1 - (2x - 5)^2$.

(i) Explain why the highest point on the curve has coordinates $\left(\frac{5}{2}, 1\right)$. [2]

(ii) Find the coordinates of the points at which the curve intersects the $x$-axis. [3]

(iii) Sketch the graph of $y = |1 - (2x - 5)^2|$, giving the coordinates of the maximum point and of the points where the curve meets the $x$-axis. [3]

(iv) Using your graph,

(a) state the number of solutions to the equation $|1 - (2x - 5)^2| = 1$, [1]

(b) find the smallest integer value of $c$ such that there are only two solutions to the equation $|1 - (2x - 5)^2| = x + c$. [1]

**– End of Paper –**