Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

MID-YEAR EXAMINATION 2018

SECONDARY 4

ADDITIONAL MATHEMATICS

4047/01

Paper 1

4 May 2018

2 hours

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number clearly on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

/ 80

This document consists of **6** printed pages.

[Turn Over

1 (i) Find the range of values of m for which the line $y = 2m - x$ intersects the curve $x^2 + y^2 - xy - 4 = 0$ at 2 distinct points. [5]

(ii) Hence, write down the values of m for which the line $y = 2m - x$ is a tangent to the curve $x^2 + y^2 - xy - 4 = 0$. [1]

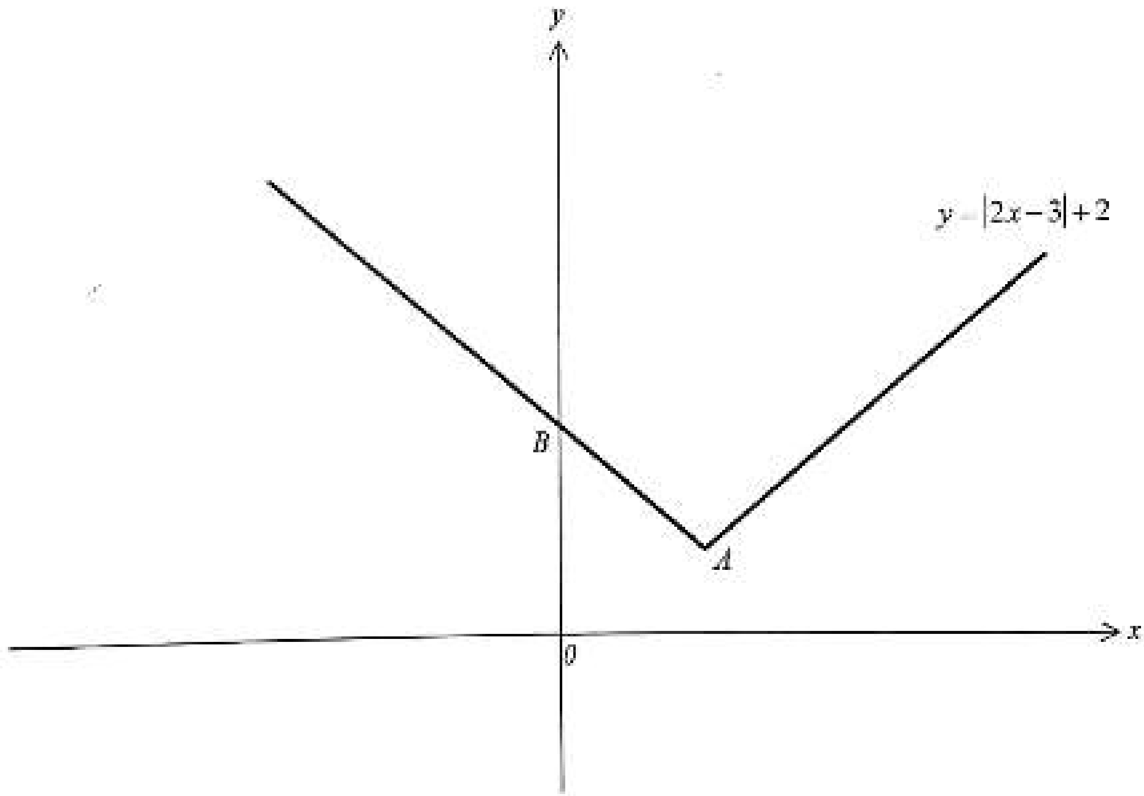
2 The line $y + x = 3$ intersects the curve $y^2 = kx$ at point $A(1, 2)$ and point B.

(i) Find the value of k. [1]

(ii) Find the coordinates of B. [3]

(iii) On the same axes, sketch the graphs of $y + x = 3$ and $y^2 = kx$. [2]

3



The diagram shows part of the graph of $y = |2x - 3| + 2$.

(i) Find the coordinates of A and of B. [2]

(ii) Solve the equation $|2x - 3| + 2 = 7$. [3]

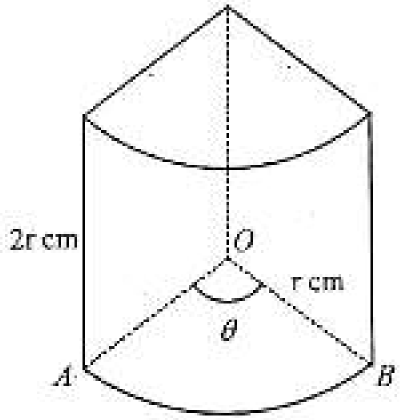
4 (i) Express $\frac{x-19}{(2x-3)(x+2)}$ in the form $\frac{A}{2x-3}+\frac{B}{x+2}$, where A and B are constants. [3]

(ii) Hence, or otherwise, find $\int \frac{3x-57}{(2x-3)(x+2)}\,dx$. [4]

5 (i) Show that $\frac{d}{dx}\left(\frac{\cos x}{2-\sin x}\right)=\frac{1-2\sin x}{(2-\sin x)^2}$. [2]

(ii) A curve is such that $\frac{dy}{dx}=\frac{2-4\sin x}{(\sin x-2)^2}$ and $(0,\ 2)$ is a point on the curve. Find the equation of the curve. [4]

6 The diagram shows a prism of height $2r$ cm made using wire. The total length of wire used is 120 cm. Its cross section is a sector OAB with centre O, radius r cm and angle AOB is θ radians .



(i) Show that $\theta=\frac{60}{r}-5$. [2]

(ii) Show that the volume, V cm^3, of the prism is given by [3]

$$V=60r^2-5r^3.$$

(iii) Given that r can vary, find the maximum value of V. [6]

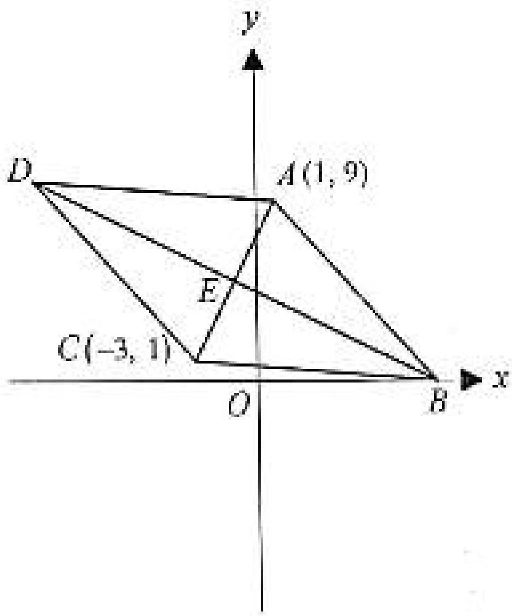
7 The difference in the height, y m, between the Changi Jetty floor level and the sea level varies with the rise and fall of tides and it can be represented mathematically by $y = -1.5\cos kt + 2.5$ where k is a constant and t is the time in hours after 12 midnight. The time between two successive high tides is 12 hours.

(i) Show that the value of k is $\frac{\pi}{6}$ radians per hour. [1]

(ii) Sketch the graph of y for $0 \le t \le 24$. [3]

(iii) A ship can berth at the Changi Jetty only if y is at least 3 m. Starting from 12 midnight, find the total length of time **(to the nearest hours and minutes)** in a day when ships can berth at the Jetty. [5]

8



The diagram shows a rhombus $ABCD$ in which the point A is $(1, 9)$, the point C is $(-3, 1)$ and the point B lies on the x-axis. The diagonals AC and BD intersect at point E.

(i) Find the coordinates of E. [2]

(ii) Show that the equation of BD is $y = -\frac{1}{2}x + 4\frac{1}{2}$. [3]

(iii) Find the coordinates of B and of D. [3]

(iv) Find the area of the rhombus $ABCD$. [2]

9 A and B are acute angles such that $\sin(A+B)=\frac{7}{8}$ and $\cos A\sin B=\frac{1}{4}$.

Without using a calculator, find the exact value of

(i) $\sin A\cos B$, [2]

(ii) $\sin(A-B)$, [2]

(iii) $\frac{\tan A}{\tan B}$. [2]

10 The equation of a curve is $y=\ln(7-2x)$ where $x<3\frac{1}{2}$.

(i) By finding an expression for $\frac{dy}{dx}$, explain why a stationary point does not exist on this curve. [3]

(ii) Find the coordinates of the point on the curve at which the normal to the curve is parallel to the line $2y-x+5=0$. [3]

11 The table shows experimental values of two variables x and y.

x	1	2	3	4	5	6
y	14	36	66	96	150	204

It is known that x and y are related by the equation $y=Ax^2-Abx$, where A and b are constants. An error is suspected to have occurred in one of the values of y.

(i) Using a scale of 2 cm to 1 unit on the horizontal axis and 2 cm to 5 units on the vertical axis, plot $\frac{y}{x}$ against x and draw a straight line graph. [3]

(ii) Hence, identify the incorrect value of y and use your graph to estimate its correct value. [2]

(iii) Estimate the values of A and of b. [3]

Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

Parent's Signature

MID-YEAR EXAMINATION 2018
SECONDARY 4

ADDITIONAL MATHEMATICS

4047/02

Paper 2

8 May 2018

2 hours 30 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number clearly on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

/ 100

This document consists of **5** printed pages and **1** blank page.

[Turn over

1 A rectangle $ABCD$ has width $AB = \left(2\sqrt{5}+3\right)$ cm and has area $\left(13\sqrt{5}+69\right)$ cm^2.

(i) Find an expression for $(AB)^2$ in the form $\left(c+d\sqrt{5}\right)$ cm^2, where c and d are integers. [2]

(ii) Find the length BC in the form $\left(p+q\sqrt{5}\right)$ cm, where p and q are integers. [3]

2 The roots of the quadratic equation $2x^2+5x+10=0$ are α and β.

(i) Find the value of $\alpha^2+\beta^2$. [3]

(ii) Show that the value of $\dfrac{\alpha^3+\beta^3}{\alpha\beta}$ is $\dfrac{35}{8}$. [2]

(iii) Find a quadratic equation whose roots are $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$. [3]

3 The equation of a curve is $y = \dfrac{e^{3x}}{4x+1}$ for $x > 0$.

(i) Find an expression for $\dfrac{dy}{dx}$. [2]

(ii) Explain why the curve has only one stationary point and why it is a minimum point. [3]

(iii) Determine the range of values of x for which y is a decreasing function. [3]

The variables x and y are such that, when $x = 3$, y is increasing at a rate of 0.9 units per second.

(iv) Find the rate of change of x when $x = 3$. [2]

4 It is given that $f(x) = x^3 + px^2 - 3x - 10$ leaves a remainder of 26 when divided by $x-3$.

(i) Find the value of p. [2]

(ii) Using your value of p in **(i)** and showing clearly your working, factorise $f(x)$. [3]

(iii) Explain why the equation $f(x) = 0$ has only one real root and state its value. [2]

(iv) Hence, solve the equation $f(x) = 10(x-2)$. [3]

5 **(a)** Find the set of values of k for which the curve $y = 2x^2 + 4kx + 10 - k$ is always positive for all real values of x. [3]

(b) Solve the equation $\log_4(x-6) = 2 - \log_4 x$. [3]

(c) Solve the equation $\log_2 x = 4 - 3\log_x 2$. [4]

6 (i) Show that $\dfrac{1+\sin 2A-\cos 2A}{1+\sin 2A+\cos 2A}=\tan A$. [3]

(ii) Hence, solve the equation $\dfrac{1+\sin 2A-\cos 2A}{1+\sin 2A+\cos 2A}=2\tan^2 A-1$ for $0° \le A \le 360°$. [4]

7 (a) Express $5^{2x}=5^{x+1}+14$ as a quadratic equation in 5^x and hence, find, correct to 2 decimal places, the value of x which satisfies the equation $5^{2x}=5^{x+1}+14$. [5]

(b) At a restaurant, soup is left to cool after removal from boiling before storage in the refrigerator. Its temperature, T°C, t minutes after removal from boiling is given by $T=30+70e^{-mt}$, where m is a constant.
It is given that the temperature of the soup is 87°C after 5 minutes.

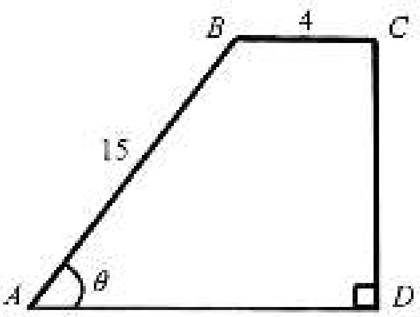
(i) Find the initial temperature of the soup. [1]

(ii) Find the value of m, correct to 3 significant figures. [2]

Soup is stored in the refrigerator only when its temperature is less than 35°C.

(iii) Determine, with working, whether the soup is ready to be stored in the refrigerator 45 minutes after removal from boiling. [2]

8



The diagram shows a fence $ABCD$ that encloses an area for crops within a garden. The fence $ABCD$ is a trapezium, where the length of AB is 15 m, BC is 4 m and angle $ADC = 90°$. The side AB makes an acute angle θ with side AD and the angle θ can vary. The perimeter of the fence $ABCD$ is L m.

(i) Show that $L=23+15\cos\theta+15\sin\theta$. [2]

(ii) Express L in the form $23+R\cos(\theta-\alpha)$, where $R>0$ and $0°<\alpha<90°$. [4]

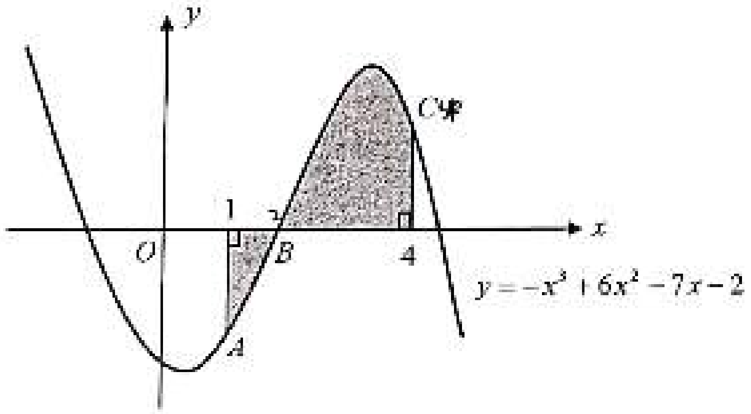
(iii) Find the greatest possible value of L and the value of θ at which this occurs. [2]

(iv) Find the value of θ for which $L = 40$ m. [3]

9 (i) Write down, and simplify, the first 4 terms in the expansion of $\left(2x-\frac{1}{x^3}\right)^8$ in descending powers of x. [3]

(ii) In the expansion of $\left(1-\frac{k}{x}\right)^2\left(2x-\frac{1}{x^3}\right)^8$, where k is a positive constant, the coefficient of x^2 is three times the coefficient of x^7. Find the value of k. [5]

10 The diagram shows part of the curve $y=-x^3+6x^2-7x-2$, where A, B and C lie on the curve. The x-coordinate of A is 1.



(i) Find the equation of the normal to the curve at A. [5]

It is given that the point B is (2, 0) and the x-coordinate of C is 4.

(ii) Find the area of the shaded region. [4]

11 The equation of a circle, C_1, is $x^2+y^2-6x-10y+9=0$ and the point $A(0, 1)$ lies on C_1.

(i) Find the radius and the coordinates of the centre of C_1. [3]

(ii) Show that the equation of the tangent to C_1 at A is $4y+3x=4$. [2]

A second circle, C_2, with centre D, passes through the points $P(-7, 5)$ and $Q(-3, 1)$. The point D also lies on the tangent in (ii).

(iii) Find the coordinates of D and the radius of C_2. [5]

(iv) Explain why the point $(-3, 4)$ lies within only one of the circles C_1 and C_2. [2]

		Working
1	(i)	$AB^2=\left(2\sqrt{5}+3\right)^2=\left(2\sqrt{5}\right)^2+2\left(2\sqrt{5}\right)(3)+3^2$ $=\left(4(5)+12\sqrt{5}+9\right)$ $=\left(29+12\sqrt{5}\right)$ cm^2
	(ii)	$BC=\frac{13\sqrt{5}+69}{2\sqrt{5}+3}$ $=\frac{13\sqrt{5}+69}{2\sqrt{5}+3}\times\frac{2\sqrt{5}-3}{2\sqrt{5}-3}$ $=\frac{26(5)-39\sqrt{5}+138\sqrt{5}-207}{4(5)-9}$ $=\frac{99\sqrt{5}-77}{11}$ $=\left(9\sqrt{5}-7\right)$ cm
2	(i)	$2x^2+5x+10=0$ has roots α and β Sum of roots $=\alpha+\beta=-\frac{(5)}{2}=-\frac{5}{2}$ Product of roots $=\alpha\beta=\frac{10}{2}=5$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $\alpha^2+\beta^2=\left(-\frac{5}{2}\right)^2-2(5)$ $=-3\frac{3}{4}$ or $-\frac{15}{4}$ or -3.75
	(ii)	$\alpha^3+\beta^3=(\alpha+\beta)[(\alpha^2+\beta^2)-\alpha\beta]$ $=\left(-\frac{5}{2}\right)\left[\left(-\frac{15}{4}\right)-5\right]$ $=21\frac{7}{8}$ $\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{21\frac{7}{8}}{5}=4\frac{3}{8}$ $=\frac{35}{8}$

		Working
	(iii)	Sum of new roots $=\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$ $=\frac{35}{8}$ Product of new roots $=\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=\alpha\beta$ $=5$ Quadratic equation is $x^2-\frac{35}{8}x+5=0$ or $8x^2-35x+40=0$
3	(i)	$y=\frac{e^{2x}}{4x+1}$ $\frac{dy}{dx}=\frac{(4x+1)2e^{2x}-e^{2x}(4)}{(4x+1)^2}$ $=\frac{2e^{2x}(4x-1)}{(4x+1)^2}$
	(ii)	Since $e^{2x}>0$ for $x>0$, there is only one stationary point where $\frac{dy}{dx}=\frac{2e^{2x}(4x-1)}{(4x+1)^2}=0$ when $4x-1=0$.

	$x=0.2$	$x=0.25$	$x=0.3$
$\frac{dy}{dx}$	negative (-0.184)	0	positive (0.151)

Using the first derivative test, the gradient changes from negative to positive, thus it is a minimum point.

(iii) For decreasing function, $e^{2x}>0$ and $(4x+1)^2>0$

$\frac{dy}{dx}=\frac{2e^{2x}(4x-1)}{(4x+1)^2}<0$

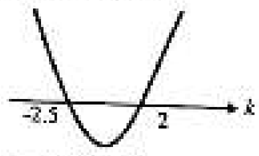
$4x-1<0$

$x<\frac{1}{4}$

Since $x>0$, $\therefore 0<x<\frac{1}{4}$.

		Working
	(iv)	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ When $x = 3$ $0.9 = \frac{e^{2(3)}[8(3)-2]}{[4(3)+1]^2} \times \frac{dx}{dt}$ $\frac{dx}{dt} = 0.9 \div \frac{e^6(22)}{(169)}$ $\frac{dx}{dt} = 0.0171$ units/s (3 sig. fig.) or $\frac{1521e^{6}}{220}$ units/s Rate of change of x is 0.0171 units/s.
4	(i)	$f(x) = x^3 + px^2 - 3x - 10$ has remainder 26 when divided by $x - 3$ Using Remainder Theorem, $f(3) = (3)^3 + p(3)^2 - 3(3) - 10 = 26$ $9p + 8 = 26$ $9p = 18$ $p = 2$
	(ii)	$f(x) = x^3 + 2x^2 - 3x - 10$ Try $x - 2$: $f(2) = (2)^3 + 2(2)^2 - 3(2) - 10 = 0$ Since $f(2) = 0$, $x - 2$ is a factor. Using Long Division (or by inspection) $x-2 \overline{)x^3+2x^2-3x-10}$, quotient x^2+4x+5 $-(x^3-2x^2)$ $4x^2-3x$ $-(4x^2-8x)$ $5x-10$ $-(5x-10)$ 0 $f(x) = (x-2)(x^2+4x+5)$

	Working
(iii)	$f(x) = (x-2)(x^2+4x+5) = 0$ Consider $x^2 + 4x + 5$, Discriminant $= (4)^2 - 4(1)(5) = -4 < 0$ $x^2 + 4x + 5$ has no real roots. $\therefore$ f(x) only has one real root $x = 2$.
(iv)	$(x-2)(x^2+4x+5) = 10(x-2)$ $(x-2)(x^2+4x+5-10) = 0$ $(x-2)(x^2+4x-5) = 0$ $(x-2)(x-1)(x+5) = 0$ $x-2=0$ or $x-1=0$ or $x+5=0$ $x=2$ or $x=1$ or $x=-5$

		Working
5	(a)	$y = 2x^2 + 4kx + 10 - k$ is positive for all real values of x.
		discriminant < 0
		$(4k)^2 - 4(2)(10-k) < 0$
		$16k^2 + 8k - 80 < 0$
		$2k^2 + k - 10 < 0$
		$(2k+5)(k-2) < 0$
		
		$\therefore\ -2.5 < k < 2$
	(b)	$\log_4(x-6) = 2 - \log_4 x$
		$\log_4(x-6) + \log_4 x = \log_4 16$
		$\log_4 x(x-6) = \log_4 16$
		$x(x-6) = 16$
		$x^2 - 6x - 16 = 0$
		$(x-8)(x+2) = 0$
		$\therefore x = 8$ or $x = -2$ (rejected, as $\log_4 -2$ is undefined.)
	(c)	$\log_2 x = 4 - 3\log_x 2$
		$\log_2 x = 4 - 3\left(\dfrac{\log_2 2}{\log_2 x}\right)$
		$(\log_2 x)^2 = 4\log_2 x - 3$
		Let $u = \log_2 x$
		$u^2 - 4u + 3 = 0$
		$(u-1)(u-3) = 0$
		$\log_2 x = 1$ or $\log_2 x = 3$
		$x = 2$ or $x = 8$

		Working
6	(i)	$\text{LHS} = \dfrac{1+\sin 2A - \cos 2A}{1+\sin 2A + \cos 2A}$
		$= \dfrac{1 + 2\sin A\cos A - (1 - 2\sin^2 A)}{1 + 2\sin A\cos A + (2\cos^2 A - 1)}$
		$= \dfrac{2\sin^2 A + 2\sin A\cos A}{2\cos^2 A + 2\sin A\cos A}$
		$= \dfrac{\sin A(\sin A + \cos A)}{\cos A(\cos A + \sin A)}$
		$= \dfrac{\sin A}{\cos A}$
		$= \tan A$ (RHS) (shown)
	(ii)	$\dfrac{1+\sin 2A - \cos 2A}{1+\sin 2A + \cos 2A} = 2\tan^2 A - 1$
		$\tan A = 2\tan^2 A - 1$
		$2\tan^2 A - \tan A - 1 = 0$
		$(2\tan A + 1)(\tan A - 1) = 0$
		$\tan A = -\dfrac{1}{2}$ or $\tan A = 1$
		For $\tan A = -\dfrac{1}{2}$, basic angle of $A = \tan^{-1}\left(\dfrac{1}{2}\right) = 26.5650...°$,
		in 2nd and 4th quadrant.
		$A = 180 - 26.5650,\ 360 - 26.5650$
		$A = 153.4°,\ 333.4°$ (to 1 d.p.)
		For $\tan A = 1$, basic angle of $A = 45°$,
		in 1st and 3rd quadrant.
		$A = 45°,\ 180+45°$
		$A = 45°,\ 225°$
		$\therefore A = 45°,\ 153.4°,\ 225°,\ 333.4°$

Working

7 (a) $5^{2x} = 5^{x+1} + 14$

Let $u = 5^x$.

$(5^x)^2 = 5(5^x) + 14$

$u^2 - 5u - 14 = 0$

$(u-7)(u+2) = 0$

$u = 7$ or $u = -2$ (rejected, as $5^x > 0$)

$5^x = 7$

$x \ln 5 = \ln 7$

$x = \dfrac{\ln 7}{\ln 5}$

$x = 1.21$ (to 2 decimal places)

(b)(i) When $t = 0$, $T = 30 + 70e^0 = 100°C$

(ii) When $t = 5$, $87 = 30 + 70e^{-m(5)}$

$70e^{-5m} = 57$

$e^{-5m} = \dfrac{57}{70}$

$-5m = \ln\left(\dfrac{57}{70}\right)$

$m = 0.041088$

$m = 0.0411$ (to 3 sig. fig.)

(iii) At $t = 45$

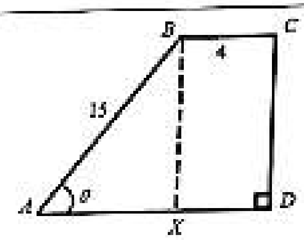
$T = 30 + 70e^{-0.041088(45)}$

$= 41.018°C > 35°C$

Since the temperature is above 35°C, the soup is not ready to be stored in the refrigerator after 45 minutes.

Working

8 (i)



Let X be vertically below B.

$AX = 15\cos\theta$ and $BX = 15\sin\theta = CD$

$L = 15 + 4 + 15\sin\theta + 4 + 15\cos\theta$

$L = 23 + 15\sin\theta + 15\cos\theta$.

(ii) $R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

Comparing coefficients:

$R\cos\alpha = 15$

$R\sin\alpha = 15$

$\tan\alpha = \dfrac{15}{15}$

$\alpha = \tan^{-1}(1) = 45°$

$R^2 = 15^2 + 15^2$

$R = \sqrt{15^2 + 15^2}$

$R = 15\sqrt{2}$

$L = 23 + 15\sqrt{2}\cos(\theta - 45°)$

(iii) Maximum value of $L = 23 + 15\sqrt{2} = 44.2$ (to 3 sig. fig.)

Occurs when $\cos(\theta - 45°) = 1$ and $\theta = 45°$.

(iv) $40 = 23 + 15\sqrt{2}\cos(\theta - 45°)$

$\cos(\theta - 45°) = \dfrac{17}{15\sqrt{2}}$

Basic angle $= \cos^{-1}\left(\dfrac{17}{15\sqrt{2}}\right) = 36.73717°$

$\theta - 45° = 36.73717°$ or $\theta - 45° = 360 - 36.73717°$

$\theta = 36.73717° + 45°$ or $\theta = (323.26283° + 45°) - 360°$

$\theta = 8.3°$ or $81.7°$ (to 1 d.p.)

		Working
9	(i)	$\left(2x-\frac{1}{x^3}\right)^8=(2x)^8+8(2x)^7\left(-\frac{1}{x^3}\right)+28(2x)^6\left(-\frac{1}{x^3}\right)^2$
		$+56(2x)^5\left(-\frac{1}{x^3}\right)^3+\cdots$
		$\left(2x-\frac{1}{x^3}\right)^8=256x^8-1024x^4+1792-\frac{1792}{x^4}+\cdots$
	(ii)	$\left(1-\frac{k}{x}\right)^2\left(2x-\frac{1}{x^3}\right)^8$
		$=\left(1-\frac{2k}{x}+\frac{k^2}{x^2}\right)\left(256x^8-1024x^4+1792-\frac{1792}{x^4}+\cdots\right)$
		Term in $x^7=\left(-\frac{2k}{x}\right)(256x^8)=-512kx^7$
		Term in $x^2=\left(\frac{k^2}{x^2}\right)(-1024x^4)=-1024k^2x^2$
		Comparing coefficients:
		$3(-512k)=-1024k^2$
		$1024k^2-1536k=0$
		$512k(2k-3)=0$
		$k=0$ or $k=\frac{3}{2}$
		$\therefore k=\frac{3}{2}$, reject $k=0$ since $k>0$.

		Working
10	(i)	$y=-x^3+6x^2-7x-2$
		$\frac{dy}{dx}=-3x^2+12x-7$
		When $x=1$, $\frac{dy}{dx}=-3(1)^2+12(1)-7=2$
		Gradient of normal $=-\frac{1}{2}$
		$y=-(1)^3+6(1)^2-7(1)-2=-4$
		Equation of normal
		$y-(-4)=-\frac{1}{2}(x-1)$
		$y=-\frac{x}{2}-3\frac{1}{2}$ or $2y+x+7=0$
	(ii)	Area of shaded region
		$=\int_2^4 -x^3+6x^2-7x-2\,dx+\left\lvert\int_1^2 -x^3+6x^2-7x-2\,dx\right\rvert$
		$=\left[-\frac{x^4}{4}+\frac{6x^3}{3}-\frac{7x^2}{2}-2x\right]_2^4+\left\lvert\left[-\frac{x^4}{4}+\frac{6x^3}{3}-\frac{7x^2}{2}-2x\right]_1^2\right\rvert$
		$=\left[-\frac{(4)^4}{4}+2(4)^3-\frac{7(4)^2}{2}-2(4)-\left(-\frac{(2)^4}{4}+2(2)^3-\frac{7(2)^2}{2}-2(2)\right)\right]$
		$+\left\lvert-\frac{(2)^4}{4}+2(2)^3-\frac{7(2)^2}{2}-2(2)-\left(-\frac{(1)^4}{4}+2(1)^3-\frac{7(1)^2}{2}-2(1)\right)\right\rvert$
		$=6+2.25$
		$=8\frac{1}{4}$ units^2

Working

11 (i)

$x^2 + y^2 - 6x - 10y + 9 = 0$

$x^2 - 6x + 9 - 9 + y^2 - 10y + 25 - 25 + 9 = 0$

$(x-3)^2 + (y-5)^2 = 5^2$

Centre of $C_1 = (3, 5)$ and radius = 5 units

Alternative

$2g = -6 \quad 2f = -10 \quad r = \sqrt{(-3)^2 + (-5)^2 - (9)} = 5$

$g = -3 \quad f = -5$

Centre = (3, 5) and radius = 5 units

(ii) Tangent through $A(0,1)$:

Gradient of radius $= \frac{1-5}{0-3}$

$= \frac{4}{3}$

Gradient of tangent $= -\frac{3}{4}$

Equation of tangent: $y - 1 = -\frac{3}{4}(x - 0)$

$y = -\frac{3}{4}x + 1$

$4y + 3x = 4$ (shown)

(iii) Mid-point of $PQ = \left(\frac{-7-3}{2}, \frac{5+1}{2}\right) = (-5, 3)$

Gradient of $PQ = \frac{5-1}{-7-(-3)} = -1$

Gradient of perpendicular bisector to chord = 1

Equation of perpendicular bisector

$y - 3 = 1(x - (-5))$

$y = x + 8$

To find D,

$-\frac{3}{4}x + 1 = x + 8$

Working

$-\frac{7}{4}x = 7$

$x = -4$

$y = -4 + 8 = 4$

D is $(-4, 4)$

Radius $= \sqrt{(-7-(-4))^2 + (5-4)^2} = \sqrt{10}$ units or 3.16 (3 sig. fig.)

(iv) Distance of $(-3, 4)$ from centre of C_1

$= \sqrt{(3-(-3))^2 + (5-4)^2}$

$= \sqrt{37} > 5$

Distance of $(-3, 4)$ from centre of C_2

$= \sqrt{(-4-(-3))^2 + (4-4)^2}$

$= \sqrt{1} < \sqrt{10}$

Since the distance to the centre is less than the radius of C_2 but it is more than the radius of C_1, the point $(-3,4)$ lies inside C_2 only and not in C_1.