



**COMMONWEALTH SECONDARY SCHOOL
MID-YEAR EXAMINATION**

ADDITIONAL MATHEMATICS

Name: _____ () Class: _____

SECONDARY THREE EXPRESS

Thursday 10 May 2018

4047/1

**1100 – 1300
2h**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Write your answers on the separate writing paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

Name of setter: Mr Toinh Long Teng

This paper consists of **5** printed pages including the cover page.

[Turn over

1 Without using a calculator, solve, for x and y , the simultaneous equations

$$27^x \div \frac{1}{3^{y+1}} = 1,$$

$$8^x \times 4^y = 16. \quad [5]$$

2 Solve the equation $\sqrt{3 - \frac{1}{2}x} = x - 3$. [5]

3 Show that the equation $kx^2 + kx + 3x + 1 = 0$ always has two real and distinct roots for all real values of k . [4]

4 (i) By using long division, divide $2x^3 - x^2 - 8x + 4$ by $x - 2$. [1]

(ii) Express $\frac{7x - 26}{2x^3 - x^2 - 8x + 4}$ as a sum of three fractions. [5]

5 (i) Expand and simplify $(\sqrt{\alpha} + \sqrt{\beta})^2$. [1]

(ii) Hence, given that the roots of $x^2 - 12x + 4 = 0$ are α and β , find a quadratic equation with integer coefficients whose roots are $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$. [5]

6 **Calculators are not allowed for this question.**

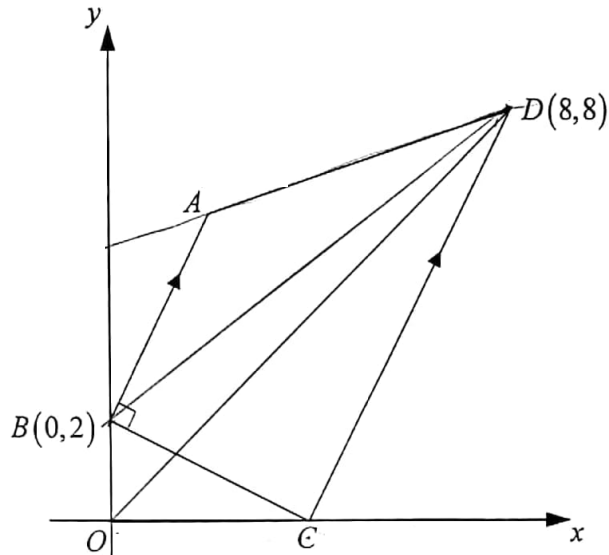
(a) A cuboid has a square base of side $\sqrt{2} + \sqrt{3}$ m and volume of $8 - \sqrt{6}$ m³. Find the height of the cuboid. [4]

(b) (i) Factorise $p^3 + q^3$. [1]

(ii) Hence express $\frac{6^{\frac{3}{2}} + 2^{\frac{3}{2}}}{6^{\frac{1}{2}} + 2^{\frac{1}{2}}}$ in the form $a - b\sqrt{c}$, where a , b and c are integers, simplifying your answers as far as possible. [3]

- 7 $x^2 - 4x + 3$ is a factor of $f(x) = x^3 - ax^2 + 7x - b$, where a and b are real constants. [3]
- (i) Find the values of a and of b . [1]
- The remainder when $f(x)$ is divided by $x - k$ is 9.
- (ii) Show that $k^3 - 5k^2 + 7k - 12 = 0$. [1]
- (iii) Solve for the value of k . [4]
- 8 (a) Given that $\log_y(x+2) = \log_y(x-4) + 1$, express x in terms of y . [3]
- (b) Solve the equation $\log_3 x - 1 = \log_x 9$. [5]
- (c) Sketch the graph of $y = \log_4 x$, indicating axial intercepts, if any. [2]
- 9 (a) Determine the range of values of p for $px^2 + 7 > 4x - 2p$ for all real values of x . [3]
- (b) The line $y = -mx + c$ is tangent to the curve $y = (x-1)^2 + 3 + c$ at the point P , where m and c are constants.
- (i) Find the positive value of m . [4]
- (ii) Using this value of m , and given that the curve passes through $(-1, 10)$, find the coordinates of P . [4]
- 10 (a) By using a suitable substitution, solve the equation $e^{2x+1} = 12e^x + 10$. [4]
- (b) John decided to start training for his shuttle run at the start of 2018. The time taken, T seconds, for him to complete the shuttle run can be modelled by the equation $T = ae^{bx} + c$, where x is the number of months from 1st January 2018 and a , b and c are constants.
- (i) State John's initial timing, in terms of a and or b and or c [1]
- (ii) State, with reasons, whether b should be positive or negative. [2]
- (iii) Hence, write down, in terms of a , b and/or c , John's timing in the long run. [1]

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The diagram shows a right-angled trapezium $ABCD$ in which the coordinates of B and D are $(0, 2)$ and $(8, 8)$ respectively. AB is parallel to DC . Points A and D lie on the line $3y - x = 16$ while points C and D lie on the line $y = 2x - 8$.

- (i) Show that the coordinates of A are $(2, 6)$. [2]
- (ii) State the angle the line segment OD makes with the positive x -axis. [1]
- (iii) Show that the perpendicular bisector of CD passes through the point A . [4]
- (iv) Find the area of triangle ABD . [2]

END OF PAPER