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GAN ENG SENG SCHOOL
Mid Year Examination 2018

GESS

ADDITIONAL MATHEMATICS
Paper 1

4047/01

8 May 2018
2 hours

Sec 4 Express/ 5 Normal (Academic)

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

	For Examiner's Use
Total	80

Answer all questions

1 The curve $x^2 + xy - y^2 = 1$ intersects the straight line $x + 2y = 3$ at the points A and B.
Find the equation of the perpendicular bisector of AB. [5]

2 (a) It is given that $-3 \le x \le 2$ is the solution of $x^2 + ax \le b$, find the value of a and of b. [2]

(b) Find the smallest value of integer b for which $-4x^2 + bx - 3$ is negative for all values of x. [3]

3 (i) Prove that $\sin^2 2\theta(\cot^2\theta - \tan^2\theta) = 4\cos 2\theta$. [3]

(ii) Hence, find, for $0^\circ < \theta < 360^\circ$ the values of for which equation $\sin^2 2\theta\left(\cot^2\theta - \tan^2\theta\right) = 2$. [3]

(iii) Find the exact value of $2\sin^2 30^\circ\left(\cot^2 15^\circ - \tan^2 15^\circ\right)$. [2]

4 (a) Solve the equation $\lg(x+14) - \lg(x-2) = 1 + 2\lg 5$, giving your answers to 2 significant figures. [3]

(b) Solve the equation $e^x - 1 - 6e^{-x} = 0$. [3]

5 Given that $\cot\theta = -k$, $90^\circ < \theta < 180^\circ$, express each of the following in terms of k.

(i) $\sin\theta$ [2]

(ii) $\operatorname{cosec}\left(90^\circ - \theta\right)$ [2]

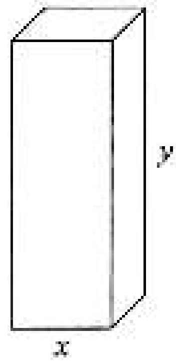
6 A curve is such that $\frac{d^2y}{dx^2} = \frac{6}{(2x-5)^2}$.

The equation of the tangent to the curve at the point $(3,-1)$ is $y - 2x = 7$

(i) Find an expression for $\frac{dy}{dx}$. [2]

(ii) Find the equation of the curve. [7]

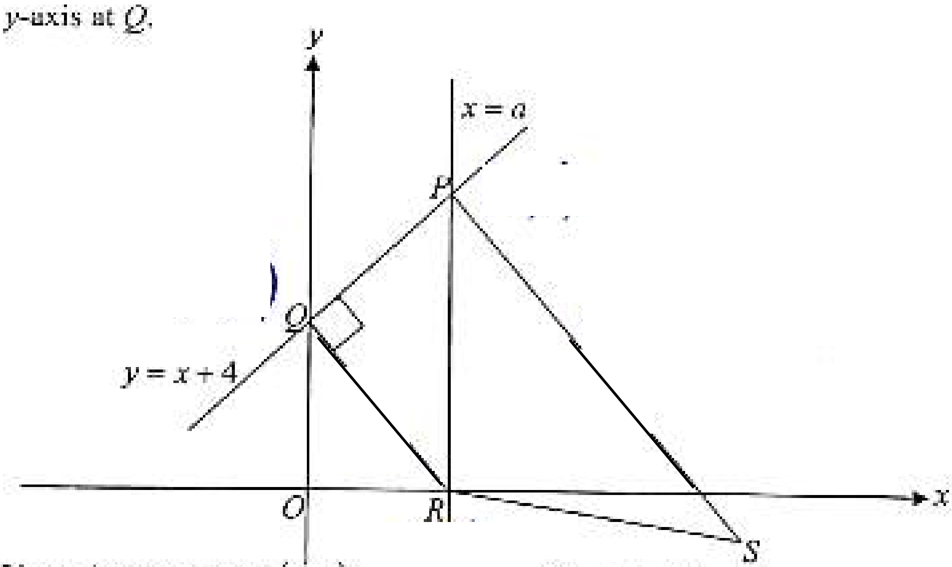
7 A closed box with square base of x m and a height of y m has a volume of 2000 m^3. The materials for the top and bottom of the box cost \$3 per square metre and the material for the sides of the box cost \$1.50 per square metre.



(i) Show that the total cost, \$ C, of the material is given by $C = 6x^2 + \frac{12000}{x}$. [3]

(ii) Given that x can vary, determine the dimensions of the box if the cost of the material is to be at a minimum. Explain clearly why this value of x gives the minimum cost of the material. [5]

8 The diagram shows the straight line $y = x + 4$ intersecting the line $x = a$ at P, and the y-axis at Q.



Given that point R is $(a, 0)$ and $\angle PQR = 90°$, calculate

(i) the value of a, [2]

(ii) the coordinates of P, [1]

(iii) the coordinates of S if $PQRS$ is a trapezium with PS parallel to QR and $PS = 5\sqrt{2}$ units, [4]

(iv) the area of trapezium $PQRS$. [2]

9 A blown glass at 516 °C is transferred to an annealing oven where it is cooled down over time. It is observed that its temperature, x°C, t hours after removal from the oven, is given by $x = Ae^{-kt} + 28$, where A and k are constants.

(i) Find the value of A. **[1]**

(ii) Find the value of k, given that the temperature of the blown glass is 35°C after 10 hours. **[2]**

(iii) Sketch the graph of x against t. **[2]**

(iv) Explain why the temperature of the blown glass will always be above 28°C. **[1]**

10 (i) Sketch the graph of $y = 1 + |x - 3|$. **[2]**

A line $y = mx + 1$ is drawn on the same axes with the graph $y = 1 + |x - 3|$.

(ii) In the case where $m = 2$, find the coordinates of the point of intersection of the line and the graph of $y = 1 + |x - 3|$. **[2]**

(iii) Determine the set of values of m for which the line will intersect the graph of $y = 1 + |x - 3|$ at exactly 2 points. **[1]**

11 (a) Find the following.

(i) $\int \left(2e^{-x} - \sqrt{e^x}\right)^2 dx$ **[3]**

(ii) $\int \frac{x+3}{x+1}\, dx$ **[3]**

(b) Given that $\int_{\frac{\pi}{12}}^{\pi} f(x)dx = 10$, find the value of m, for which $\int_{\frac{\pi}{12}}^{\pi} [f(x) - m\cos 2x]dx = 3$. **[3]**

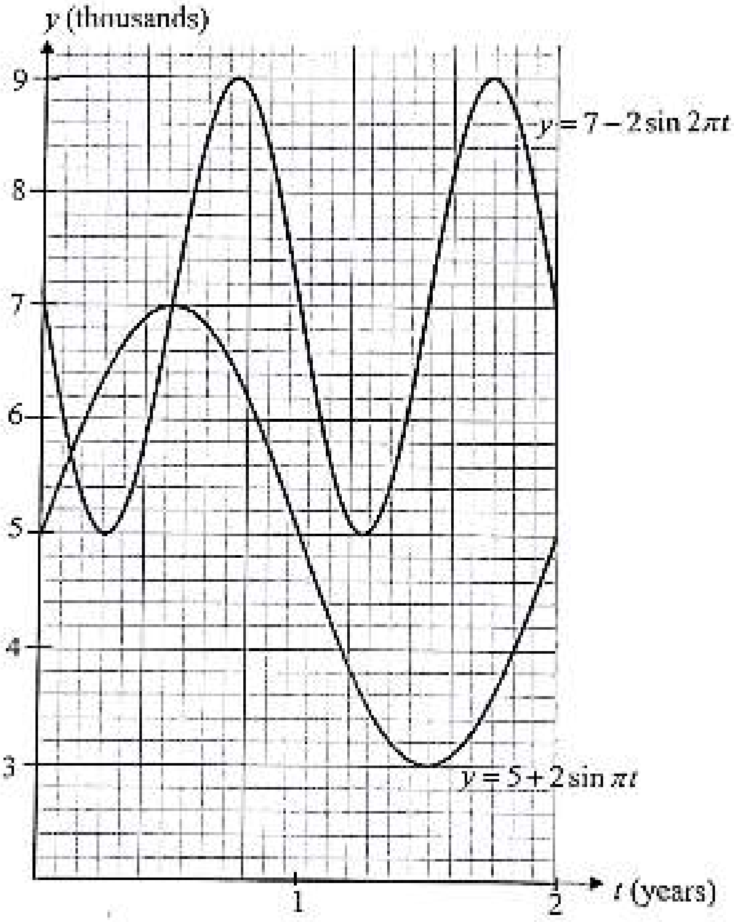
12 The population (in thousands) of two different types of insects in a garden can be modelled by the following functions:

Butterflies: $y = 5 + 2\sin \pi t$

Grasshoppers: $y = 7 - 2\sin 2\pi t$

where t is the number of years from the start when the populations were being measured.

The diagram shows part of the graph of $y = 5 + 2\sin \pi t$ and $y = 7 - 2\sin 2\pi t$ for the first 2 years.



(i) State the amplitude of $y = 5 + 2\sin \pi t$ and the period of $y = 7 - 2\sin 2\pi t$. **[2]**

(ii) Solve for t, the number of years it takes for the population of grasshoppers to first reach 8500. **[3]**

(iii) Find the number of times over a 10 year period where the population of butterflies reaches its maximum value. **[1]**

End of paper

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GAN ENG SENG SCHOOL
Mid-Year Examination 2018

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS
Paper 2

4047/02

4 May 2018
2 hours 30 minutes

Sec 4 Express/ 5 Normal (Academic)

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

	For Examiner's Use
Total	/ 100

This paper consists of 6 printed pages including the cover page. **[Turn over]**

Answer ALL questions

1. The remainders when the expression $4x^3 + ax^2 + bx - 6$ is divided by $(x-2)$ and $(x-1)$ are 12 and -5 respectively.

   (i) Show that the remainder when $4x^3 + ax^2 + bx - 6$ is divided by $(x+2)$ is -56. [4]

   (ii) Show that $4x^3 + ax^2 + bx - 6 = 0$ has only one real root. [4]

2. The roots of a quadratic equation $2x^2 - 6x + 3 = 0$ are $2\alpha + \beta$ and $\alpha + 2\beta$.

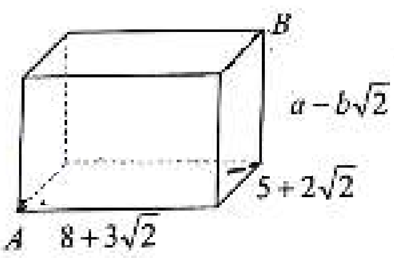
   (i) Show that $\alpha^2 + \beta^2 = 2$. [4]

   (ii) Find the quadratic equation whose roots are $\dfrac{2}{\alpha^3}$ and $\dfrac{2}{\beta^3}$ [4]

3. (a) In the binomial expansion of $(1 + kx)^n$, where $n \geq 4$, the coefficient of x^2 and x^3 are equal.
   Express k in terms of n. [3]

   (b) Find the value of the term independent of x in the binomial expansion of $\left(2x - \dfrac{1}{2x}\right)^{18}$ [3]

4.

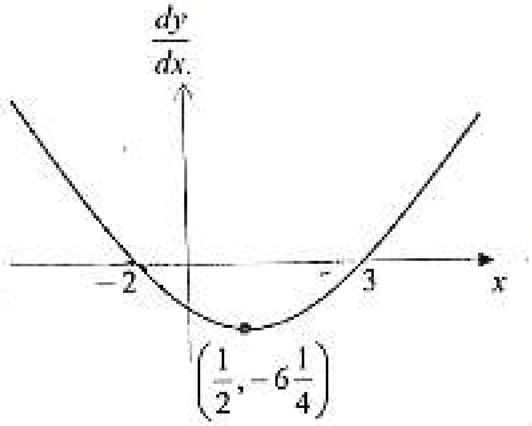


An open small fish tank is made of glass and has a length of $8 + 3\sqrt{2}$ cm, a width of $5 + 2\sqrt{2}$ cm and a height of $a - b\sqrt{2}$ cm, where a and b are positive integers. The longest rod that can be placed in the fish tank is from A to B. Given that the length of AB is $\sqrt{159 + 44\sqrt{2}}$ cm, find

(i) the value of a and the value of b, [6]

(ii) the exact volume of the fish tank. [2]

5. (i)



The diagram shows a sketch of the graph of $\frac{dy}{dx}$ against x, for a curve $y = f(x)$. The graph intersects the x-axis at -2 and 3. State the x-coordinate(s) of the stationary point(s) of the curve and the nature of the stationary point(s). Give reasons for your choice. [4]

(ii) A curve has the equation, $y = 16 - (x-2)^4$. The point (p, q) is the stationary point on the curve.

(a) Determine the value of p and of q. [3]

(b) Determine whether y is increasing or decreasing

(i) for values of x less than p, [1]

(ii) for values of x greater than p. [1]

(c) What is the value of $\frac{d^2y}{dx^2}$ at the stationary point? [2]

(d) What does the result of part (b) and (c) tell you about the nature of the stationary point? [1]

6. In the lens equation, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, u is the distance of the object from the lens, v is the distance of the image from the lens and f is the focal length of fixed distance.

(i) Find $\frac{du}{dv}$. [3]

A convex lens has focal length of 10 cm.

When an object is 25 cm from the lens, it is moving towards the lens at 0.5 cm/s.

(ii) Find the rate at which the image distance is increasing. [3]

7. Express $\dfrac{7x^2+2}{(2x-1)(x^2+1)}$ as a sum of partial fractions. [5]

8. The equation of a curve is $y = \dfrac{e^{2x}}{x+2}$ for $x > -2$.

(i) Find $\dfrac{dy}{dx}$. [2]

The curve has a turning point.

(ii) Find the coordinates of the turning point and determine its nature. [4]

The tangent at $x = 1$ meets the x-axis at B.

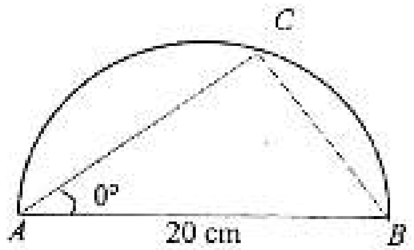
(iii) Find the x-coordinate of B. [4]

9. (i) Show that $\dfrac{d}{dx}(\ln(\sin x)) = \cot x$ [2]

(ii) Differentiate $x\cot x$ with respect to x. [2]

(iii) Use the result of part (i) and part (ii) to evaluate $\int x\operatorname{cosec}^2 x\,dx$. [3]

10.



Triangle ABC is inscribed in a semi-circle in which AB is a diameter of length 20 cm. $\angle BAC = \theta°$ and θ can vary.

(i) Show that the perimeter of triangle ABC, P cm, is $20\cos\theta + 20\sin\theta + 20$ cm. [2]

(ii) Express P in the form $R\sin(\theta+\alpha)+k$ where α is acute and R and k are constants. [3]

(iii) Find the value of θ for which P is a maximum and state the value of P. [2]

(iv) Find the value(s) of θ for which $P = 47.0$ cm. [2]

(v) Show that the area of triangle ABC, A cm^2, can be expressed in the form $A = h\sin 2\theta$ cm^2. State the value of h. [2]

11. A circle C_1, passes through the points $A(-2, -1)$ and $B(-2, 5)$. The line $x = 7$ is a tangent to the circle.

(i) Find the equation of the circle C_1. [5]

The tangent at $A(-2, -1)$ meets the tangent $x = 7$ at R.

(ii) Find the coordinates of R. [3]

(iii) Determine whether the point (6, 6) lies within or outside the circle. [2]

12. The mass, m mg, of a radioactive substance decreases with time, t hours. Measured values of m and t are given in the following table.

t (hours)	2	4	6	8	10
m mg	52.2	45.3	36.4	34.3	29.8

It is known that m and t are related by the equation $m = m_0e^{kt}$ where m_0 and k are constants. One value of m has been misread.

(i) Plot $\ln m$ against t for the given data and draw a straight line graph, [2]

(ii) Use your graph to estimate the value of k and of m_0. [2]

(iii) What is the wrong value of m and what is its estimated correct value? [2]

(iv) Estimate the number of hours for the original mass of the substance to be halved. [2]

(v) Explain whether m will reach zero after a long time. [1]

END of PAPER